PLAYING THE ODDS

Student/Class Goal
Students want to be able to make predictions in card games.

Outcome *(lesson objective)*
Students will apply prior knowledge of working with fractions and will further improve their ability to solve problems using all four basic operations on fractions. In addition, students will select the appropriate formula for contextual situations and define vocabulary dealing with probability.

Time Frame

4 Hours

Standard *Use Math to Solve Problems and Communicate*
(Primary benchmarks in bold.)

NRS EFL
5-6

Number Sense	**Benchmarks**	**Geometry & Measurement**	**Benchmarks**	**Processes**	**Benchmarks**
Words to numbers connection	4.1	Geometric figures		Word problems	5.25
Calculation	5.1	Coordinate system		Problem solving strategies	5.26
Order of operations		Perimeter/area/volume formulas		Solutions analysis	5.27
Compare/order numbers		Graphing two-dimensional figures		Calculator	6.29(optional)
Estimation		Measurement relationships		Math terminology/symbols	6.30
Exponents/radical expressions		Pythagorean theorem		Logical progression	5.30
Algebra & Patterns	**Benchmarks**	Measurement applications		Contextual situations	**5.31**
Patterns/sequences		Measurement conversions		Mathematical material	
Equations/expressions	5.16/6.16	Rounding		Logical terms	5.33
Linear/nonlinear representations		**Data Analysis & Probability**	**Benchmarks**	Accuracy/precision	
Graphing		Data interpretation		Real-life applications	**6.36**
Linear equations		Data displays construction		Independence/range/fluency	5.36
Quadratic equations		Central tendency			
		Probabilities			
		Contextual probability	**5.23, 5.24, 6.24, 6.25**		

Vocabulary
Probability: Branch of mathematics that studies not only the possible outcomes of a given event but also each outcome's relative likelihood and distribution.
Independent Events: The occurrence of one event has no effect on the probability of the occurrence of another event.
Dependent Events: The occurrence of an initial event does have an effect on the probability of a subsequent event occurring.
Conditional Probability: The probability of an event occurring with the assumption that another event has already occurred.
Permutation: A way of selecting objects out of a larger group where order **does** matter. In the formula, n is the total number of objects and k is the number you are selecting.
Combination: A way of selecting objects out of a larger group where order **does not** matter. In the formula, n is the total number of objects and k is the number you are selecting.

Materials
Breakdown of card decks—Handout
Formulas—Handout. ***Go through the formulas as you come to them in the lesson. Have students put the formulas into "English" so they know what they mean.
Calculators may be useful, but optional.

Learner Prior Knowledge
Students should be familiar with applying all four arithmetic operations on fractions.
Reducing fractions.
Function notation.

Instructional Activities
Step 1: Short review of multiplying/dividing fractions and adding/subtracting fractions regardless of whether the denominators are the same or not.
Step 2: Background Information

Probability or odds are the chances that a desired event will happen. This is given as a percentage (50% chance that flipping a coin will result in heads) or a fraction (1/2 chance). Card games offer up numerous possibilities for calculating probabilities. You'll be using three card decks, each described on one of the handouts. Give this to the students and make sure to go through it. (If you have actual decks, you could hand them out, but it would not be necessary to have physical decks.) After everyone is familiar with the three deck types, pass out the formula sheet. Explain that part of the planning step (step 2 of

Polya's process) will include deciding which formula is best for the given situation. **When you introduce a new formula, put it into words. For example, for conditional probability, have them write out something like: the probability of event A happening given the fact that event B has already happened is the probability of both events happening divided by the probability of just event A happening. It may also be a good idea to have them break it all the way down into just simple probabilities. For example, everywhere they see the independent probability formula of P(A and B), they could rewrite that as P(A)*P(B).

Step 3: <u>Simple Probability</u>

- (*I do*) As a deck of cards is something most, if not all, students will have prior familiarity with, the games used for context will be card games. In order to ensure that everyone is on the same page, it may be useful to have a deck on hand to show everyone. In place of an actual deck, a description of the deck makeup can be given (Number of suits, number of cards per suit, how many of each "type" in the entire deck—i.e. 4 total kings, 1 of each suit). We will be randomly drawing cards from this deck. Point out the formula for simple probability. Explain in detail how we know that the probability of drawing the Ace of Spades is 1 in 52 based on the formula. Do another problem, this time we want to know the probability of drawing a black-suited Ace. This will introduce the addition rule. Make sure to describe your thought process as you work through the problem.
- (*We do*) For this portion, you will want to incorporate discussion and as much input from the students as possible. This time, we will be using a *Uno* card deck. This deck consists of 108 cards instead of the 52 in a standard deck. In a discussion with the students, find the probability of drawing a green Skip card. Make sure to discuss how the denominator in the formula changes. Then, discuss the probability of drawing either a blue card or a reverse card. The discussion should include how the numerator/denominator is similar/different from other problems as well as how, because of the "or", we use the addition rule once more. As we have overlap (blue reverse cards), make sure to discuss why/how you must subtract those out.
- (*You do*) Change the deck to a Euchre deck (24 cards, see handout for card breakdown). Have the students individually find the probability of drawing a heart. Bring the class back together for discussion and then have them find the probability of drawing either a spade or a jack.

Step 4: <u>Independent/Dependent Probability</u>

Begin by having the class discuss the words "independent" and "dependent." Try to come up with a rough definition of what two *dependent* events would be and what two *independent* events would be. Eventually, we should arrive at the idea that if two events are *dependent* then their outcomes are intertwined, one depends upon the other. While two *independent* events have no effect on one another.

- (*I do*) Go back to the standard deck of cards. We are going to draw a card from the deck, replace it and shuffle the deck, and then draw a second card. The problem to pose is: what is the probability of drawing the Ace of Spades *and* the King of Clubs. Since we use the word *and* now instead of *or*, we use the multiplication rule. Explain that since the card was replaced and the deck reshuffled, it was possible that we could have drawn the Ace of Spades on the second draw even if we got it on the first draw. This makes the two events *independent*. As the two events are independent, there is no overlap in their probabilities. This means we just use the multiplicative rule and multiply the probability of drawing the Ace of Spades (1/52) by the probability of drawing the King of Clubs (1/52) to get 1/2704. This time, pose the problem of drawing two spades in a row, without replacing the first card. Explain why this is *dependent*. As a dependent event, we have to be a bit more careful. The formula changed a bit, we still take P(A), which in this case is 13/52=1/4. We multiply that by P(B|A), which just means, we draw another spade from a deck of now 51 cards. Since we already drew one spade, instead of 13 in the deck, there are 12. So we multiply by 12/51=4/17. Thus, we get 1/4*4/17=1/17.
- (*We do*) Once again, we switch to the *Uno* deck. Pose the following two problems to discuss and solve as an entire class: P(drawing a red card and a green card) with replacement [***Answer: probability = (25/108)*(25/108) = 625/11,664] and P(drawing a red and a green card) without replacement [***Answer: probability = (25/108)*(25/107) = 625/11,556]. This will allow them to see that the two probabilities are actually different.
- (*You do*) Pose the following two problems for them to do on their own with respect to the Euchre deck: P(drawing two Aces) with and without replacement. [***With Replacement Answer: probability = (4/24)*(4/24)=(1/6)*(1/6) = 1/36] and [***Without Replacement Answer: probability = (4/24)*(3/23)=(1/6)*(3/23) = 3/138=1/46].

Step 5: <u>Conditional Probability</u>

Conditional probability is an extension of the dependent events above. In order to have conditional probability, we must have a subsequent event that depends on the previous event. For example: the probability of having a car accident if you are a male. Here, the first event is choosing a random male, and the second event is the probability of having a car accident. Conditional probability uses a different formula than dependent probability, so point it out on the sheet.

- (*I do*) Go back to the standard deck of cards. We have upped the level of difficulty, so explaining your thought process is key. Once again, we will draw two cards back-to-back. We want to know the probability of drawing a spade as the second card given that our first card was also a spade. The formula breaks this down into a fraction of two probabilities we know how to find. On the top is a dependent probability as we aren't reshuffling and on the bottom is a simple probability. Make sure to show all steps and explain your reasoning throughout the problem. [Answer = 3/51]

- (*We do*) With the *Uno* deck, discuss with the students how to solve the following problem: P(drawing any type of wild|drawing a yellow card). [Answer = 8/107]
- (*You do*) Have each student solve the following problem on their own based on the Euchre deck: P(drawing a face card|drawing an ace). ***Hint: An ace is **not** considered a face card. [Answer = 12/23]

Step 6: <u>Permutations/Combinations</u>

As these are probably two new words, you will need to define them. We are no longer talking about probabilities now. Instead we are looking for amounts. One easy way to think of combinations is to think about the word combine. When you combine things, you usually just want to group them together without worrying about order. A combination for mathematics occurs when the order of the objects does not matter. A permutation, on the other hand, is what we use to describe a grouping of objects when order *does* matter.

- (*I do*) First, we want to know how many different ways can we arrange the 52 cards in the standard deck. It should be clear that moving any two cards changes the arrangement, thus order matters and we have a permutation. Explain your steps to set up the formula (n and k are both 52) but do not find the answer (it will be far too large). Then, as poker is a common game played with a 52 card deck, a possible question would be, how many possible poker hands are there? This means, how many possible five card combinations are there? Since order does not matter (for example: if you have the ace of spades, it does not matter whether that was the first card dealt to you or the fifth) we have a combination with n=52 and k=5. Again, explain how you use the formula to find the answer. [Answer = 2,598,960]
- (*We do*) Switching to the *Uno* deck, pose the same two questions: How many distinct ways can we arrange the 108 cards (n=108, k=108)? And, how many distinct 8 card hands are there (n=108, k=8)? This deck has repeating cards, so *distinct* is important. Make sure to discuss the steps with the class. (Again, do not find the permutation answer, just set up the equation. It will be too large to display on a calculator, even if you use the online one found below.) [Combination answer = 352,025,629,371]
- (*You do*) Pose the following two questions based on the Euchre deck: How many distinct ways can we arrange the 24 cards (n=24, k=24, again, just set up and do not solve)? And, how many distinct 5 card hands are there (n=24, k=5)? This will be different than the 52 card deck as we have less cards. [Answer = 42,504]

Assessment/Evidence (*based on outcome)*

Each of the *you do* steps will serve as assessment. The instructor should be able to gauge understanding by having different students provide their solutions and explanations of how they arrived at that solution. In addition, during the *we do* steps, instructors should be encouraging all students to participate in the discussion. The ability to provide input in these discussions will help the teacher gauge each student's mastery of the concepts.

Have each student pick a five-card poker hand (1 pair, 2 pairs, flush, straight, etc.) and have them find the odds of getting that dealt to them at the start. We are not concerned with what how many other players there are or what their cards may be.

Teacher Reflection/Lesson Evaluation

Not yet completed.

Next Steps

Introduce the concepts of Combinations and Permutations. So far, all of our denominators (total number of outcomes) were easily calculated. For more difficult problems, such as total number of possible poker hands, it would be too tedious to just count up the total number. This is where combinations and permutations come in.

Technology Integration

http://joemath.com/math124/Calculator/factorial.htm

Factorials, Permutations and Combinations Calculator

Purposeful/Transparent

Students want to be able to apply the concept of probability to everyday situations. Teachers will use games as a contextual example to model and guide students through the concepts of simple probability, independent/dependent probability, and conditional probability.

Contextual

While all examples given are in the context of card games, this can be used for many other contexts: rolling of dice, using a spinner, drawing objects from a bag, picking socks from a drawer, or any other scenario with random choice. However, probability extends to other real-life situations such as the weather, risk (car accidents and life insurance), and number of people at a store at a particular time of day.

Building Expertise

Students will already have knowledge of working with fractions and function notation. This lesson will allow them to combine the two concepts and use more in depth formulas.